imagine math®

FOUNDATIONS PAPER

Mathematical Discourse:

Mathematics Success Requires Fluency with the Technical Language of Mathematics and Proficiency in Mathematic Practices

Executive Summary

Imagine Math® 3+ is a research-based, standards-aligned supplemental program that provides meaningful practice and promotes mastery of grade-level mathematics content from Grade 3 to High School (including Algebra I and Geometry). Imagine Math 3+ helps students from all backgrounds develop confidence in their abilities and a love for mathematics by fostering mathematical language proficiency and discourse. Imagine Math's linguistically accurate and multi-representational learning environments are intentionally designed to support students' conceptual understanding of mathematics, and promote mathematical discourse to help students develop effective communication skills and a deep understanding of mathematics.

Mathematical Discourse

The National Council of Teachers of Mathematics (2014b) describes mathematical discourse as "the purposeful exchange of ideas through classroom discussion, as well as other forms of verbal, visual, and written communication" (p. 24) and a "primary mechanism for developing conceptual understanding" (p. 30). The Charles A. Dana Center (2019) emphasized the importance of mathematical discourse in defining mathematic rigor as "a set of skills that centers on the communication and use of mathematical language" (p. 6).

To achieve success in mathematics, "students must be able to communicate their ideas and reasoning with clarity and precision by using the appropriate mathematical symbols and terminology" (Dana Center, 2019, p. 6) as they engage in the procedures and practices of mathematics. As a foundational step, they also need to clearly understand mathematical vocabulary, become familiar with vocabulary to understand new mathematical concepts, and then use such vocabulary to reason within these new abstract conceptualizations. To communicate their own ideas, they need mathematical language to conceive and explore new concepts presented to them.

The Research

DISCOURSE

Researchers agree that discourse, language, and vocabulary play a critical role in learning mathematics (Groth, 2013; Sammons, 2018; Vukovic & Lesaux, 2013). Without access to discourse, opportunities to learn mathematics are significantly limited (Banse et al., 2016). Studies show positive associations between mathematical discourse that emphasizes reasoning and problem solving and student learning outcomes (Michaels et al., 2008). According to Smith and Stein (2018), mathematical discourse provides benefits for students across grade levels, including those with learning disabilities and struggling in mathematics. It fosters students' language development by promoting their use of words, symbols, and models to represent their mathematical thinking, make sense of their ideas, form connections across concepts, and clarify their understanding (Huinker & Bill, 2017). Opportunities like journal writing help students learn to express their understanding of vocabulary through written text. Graphic organizers help students communicate using multiple representations (e.g., equations, models, examples and non-examples). Through multimodal communication (e.g., verbal, written, pictures) students learn how to communicate clearly, while strengthening their conceptual understanding of key mathematics concepts.

MULTIPLE REPRESENTATIONS

Research recommends the use of multiple representations (contextual, visual, verbal, physical, and symbolic) to support students' understanding of concepts and procedures (NCTM, 2014b) and the use of multimodal forms of communication. Studies show positive effects of students' use of multiple representations and their conceptual understanding (Ainsworth, 2006; Rau et al., 2009). Yet, many students struggle to negotiate the different forms and functions of representations (Heinze et al., 2009). Helping students make connections across representations (e.g., diagrams, tables, models, equations, real-world situations) allows students to make explicit connections to, and fosters a deeper understanding of, the underlying mathematics concepts (Dreher et al., 2016; Duval, 2006), develops flexibility in both selection and use of tactics when solving problems (Dreher et al., 2016; Marshall et al., 2010), and teaches students to organize their ideas to communicate effectively using appropriate mathematical language (Huinker & Bill, 2017). Research suggests teaching students to use and apply the language of mathematics in oral, written, and representational forms leads to improvements in reasoning, conceptual understanding, and discourse skills (Moschkovich, 2013; Riccomini et al., 2015).

VOCABULARY

Mathematics vocabulary is an important component of instruction and mathematical discourse (Lin et al., 2021; Seethaler et al., 2011) and can predict students' performance in mathematics (van der Walt, 2009). Learning mathematics implies learning a new language. Engaging in mathematical activities necessitates fluency in the mathematical language. A significant portion of mathematical activity is learning and skillfully using mathematical language by diving into new definitions, and using these definitions and their related symbols. As students become familiar with mathematic vocabulary, they start to reason, explore theorems, and so on. Learning to communicate effectively "through the language of mathematics requires a robust vocabulary knowledge base; flexibility; fluency and proficiency with numbers, symbols, words, and diagrams" (Riccomini et al., 2015, p. 237).

The relationship between discourse and vocabulary is reciprocal; vocabulary learning is necessary for engaging in mathematical discourse, and mathematical discourse supports the acquisition of content-specific vocabulary. Mathematical discourse supports vocabulary development for all students by activating students' prior knowledge and connecting it with new vocabulary words (Riccomini et al., 2015). It presents and models word meanings using relevant contexts, and provides repeated and meaningful opportunities to apply the meaning of new words (Bay-Williams & Liver, 2009). Sentence starters, graphic organizers, word walls, and personal glossaries help scaffold students' understanding and new vocabulary use.

To engage students in using mathematical discourse and vocabulary, online learning environments should actively engage students in the learning process by incorporating talk moves, such as probing and purposeful questions (e.g., how and why, "What strategy might you use to solve this problem?") that ask students to explain, elaborate, or clarify their understanding of mathematics concepts, and specific questions (e.g., explicit and direct) that draw attention to critical mathematics content and scaffold learning (Banse et al., 2016), or provide concrete support for students who are struggling ("Could you draw a picture to help you solve this problem?") (Harbour & Denham, 2021). With consistent opportunities to strengthen mathematical discourse practices, language development, and vocabulary, students' communication deepens, broadens, and becomes increasingly complex.

How Imagine Math 3+ Integrated Research into Practice

Imagine Math 3+ fosters mathematical discourse in multiple ways. The program engages students in tasks that promote the use of multiple representations, asks high-level questions to support students' acquisition and use of precise mathematics language, and promotes mathematical discourse by prompting students to explain their strategies and solutions, connect prior knowledge to new concepts, clarify understanding, and communicate their thinking effectively. Application tasks within the program integrate these tasks to fully support mathematical discourse. To illustrate, in a fifth-grade Application Task, "Build a Tropical Rainforest Greenhouse," students are asked how they could use multiplication to determine the volume of four pools in a rainforest greenhouse (Figure 1). The open-ended nature of this Application Task allows students to employ diverse strategies to determine the volume of the pools they design. Students make connections across different concepts (measurement, geometry, whole-number operations) and express their thinking using multiple modalities (e.g., completing a table with the appropriate measurements and providing a written explanation of the strategy they used to solve the problem). The Application Task provides a glossary of academic (e.g., greenhouse) and mathematics vocabulary (e.g., cubic unit). Students are encouraged to use these words in response to written prompts such as, "How did you find the length and width for each pool?" There are also opportunities to engage in discourse in the "Talk About It" sections. Students are invited to make their mathematical thinking visible by collaborating with their peers to design different floorplans for a greenhouse, comparing their designs, identifying patterns in their measurements, and redesigning a floorplan with different dimensions as an extension.

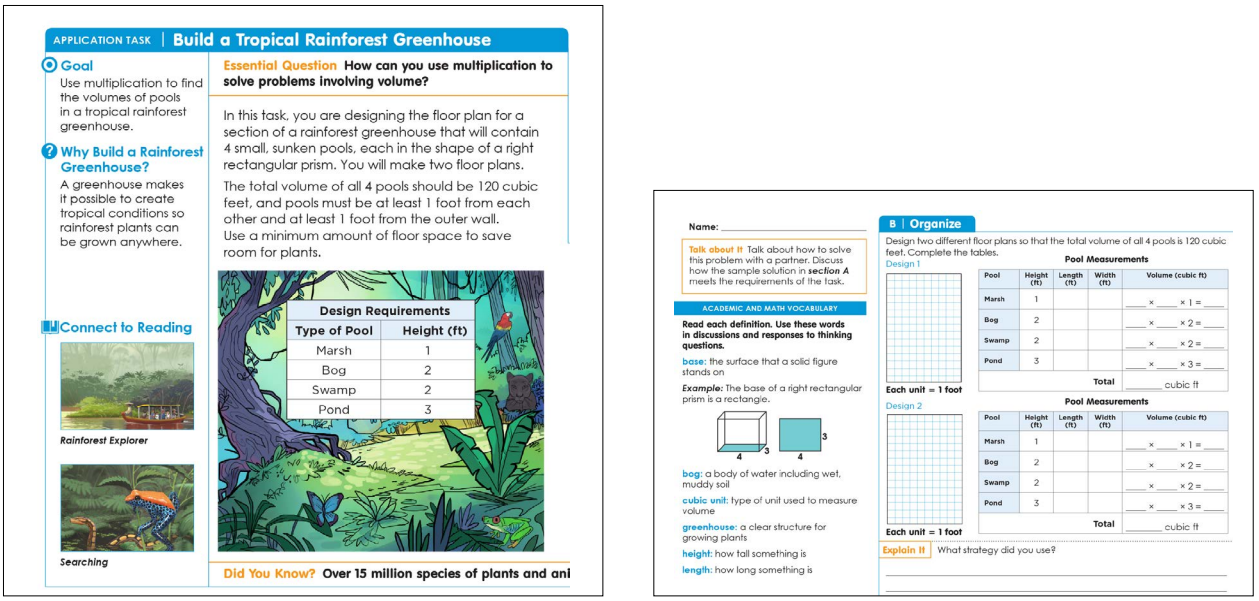

Figure 1. *Build a Tropical Rainforest Greenhouse, Grade 5*

In addition to Application Tasks, Imagine Math 3+ includes access to Live Teachers, who promote mathematical discourse during interactions with students. During a lesson, students can access one of Imaging Math's Live Teachers for intensive, one-on-one support. During this time, the teacher and student can communicate using voice, chat, and a two-way interactive whiteboard. The Live Teacher can view the students' work to pinpoint areas of difficulty and pose purposeful questions to encourage students to explain their thought processes. For instance, rather than asking a question that elicits a single-word answer, "I ate 10% of my pizza and you ate 25% of your pizza. If our pizzas were the same size, who ate more?" teachers ask questions like, "If I ate 10% of a pizza and you ate 25% of a pizza, what is one way I could have eaten more pizza than you? Draw a picture and write an equation to justify your thinking." The transcript below (Figure 2) illustrates a conversation between a real student and a Live Teacher. Notice how the teacher elicits the student's thinking using probing questions to encourage predictions, explanations, and justifications, rather than proposing their use of a specific procedure to solve the problem. These teachers are trained to facilitate productive mathematical discourse, which includes the use of appropriate talk moves and rich questioning techniques to help guide students toward a conceptual understanding of the content.

Teacher: What do you wonder about what I wrote?
Student: Why is there 20 ounces of chicken? Did you can it? How does this relate to the question?
Teacher: Great questions. This is an example that will help you understand how the numbers are related. How many servings do you predict I can make?
Student: It depends...how many ounces are in a serving?
Teacher: You are asking great questions. Suppose I want 4 ounces in each serving.
Student: Okay, you can make 5 servings.
Teacher: Terrific! How did you figure that out?
Student: 4 x 5 equals 20 and you had 4 ounces of chicken in each serving.
Teacher: How else can we figure out how many servings we can make from 20 ounces of chicken?
Student: 20 divided by 5 equals 4.

Figure 2. *Live Teacher and student chat*

In addition to interacting with certified Live Teachers in using mathematical discourse, Imagine Math 3+ students receive Journaling Pages, which they can print and use during any lesson (Figure 4). These resources are available in English and Spanish. They provide space for students to organize their mathematical thinking, take notes on important concepts, write their own definitions of key vocabulary words, solve problems using various strategies, ask questions, and reflect on their learning. In one section of the Journaling Pages, students are encouraged to reflect on the lesson and "Write at least one important math vocabulary word or phrase that was used in this lesson. For each word or phrase, write the definition in your own words and draw a visual representation." Students can record their thoughts and responses in their native language, along with a visual representation, to promote stronger language development.

Imagine Math 3+ further promotes language development by encouraging the use of multimodal forms of communication (e.g., written and oral language, symbols, models, virtual manipulatives). This helps students build meaning, clarify understanding, compare strategies, and form connections across concepts and representations. In Figure 3, students solve the problem, "Bottled water comes in cases of 24 bottles. You need 150 bottles of water for a school event. How many cases of water do you need to buy?" Imagine Math 3+ recommends several different strategies ("Draw a picture, diagram or model" or "Look for a pattern"), which illustrate the number of ways a student can solve the problem. Six digital manipulatives are also available for students to use (bar models, number lines, fraction pieces, fraction shapes, area models, base-ten manipulatives). "Write or Talk" prompts are embedded in the lesson to encourage students to reflect on their problem solving. These prompts ask students to explain their strategy and the reasonableness of their answer. Sentence frames are provided to scaffold students' responses ("An important part of the solution to the problem is…" or "This strategy was effective for the problem because…"). Linking words and phrases ("To give an example" or "In contrast") are also included, to enhance a student's ability to communicate clearly and coherently.

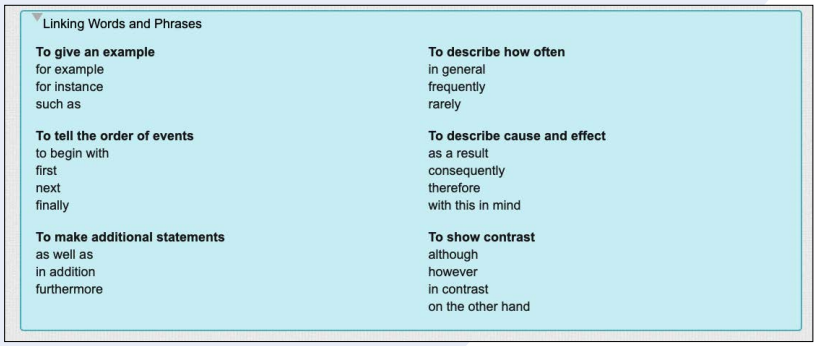

Figure 3. *Use of multimodal forms of communication, Grade 5*

Imagine Math recognizes the importance of vocabulary development. The program supports students' acquisition and use of vocabulary by helping them connect prior knowledge to new vocabulary words, providing real-world contexts, modeling word meanings, and providing repeated exposure to new words. Students have access to an interactive glossary of more than 500 essential academic and mathematics vocabulary words, which are available in English, Spanish, Tagalog, Haitian-Creole, Arabic, and Vietnamese. They have the option to interact with the interface, and can opt to have the words read aloud to them in their chosen language. This glossary strengthens students' vocabulary knowledge by ensuring the definitions, models, and problems are defined accurately and consistently. The glossary also provides visual supports and examples to help students encode meaning. For example, the term "equivalent fractions" provides a written definition, an area model, symbol notation, and a number line in the glossary (Figure 5). The use of multiple representations helps students develop a conceptual understanding of the concept and learn to translate across different representations.

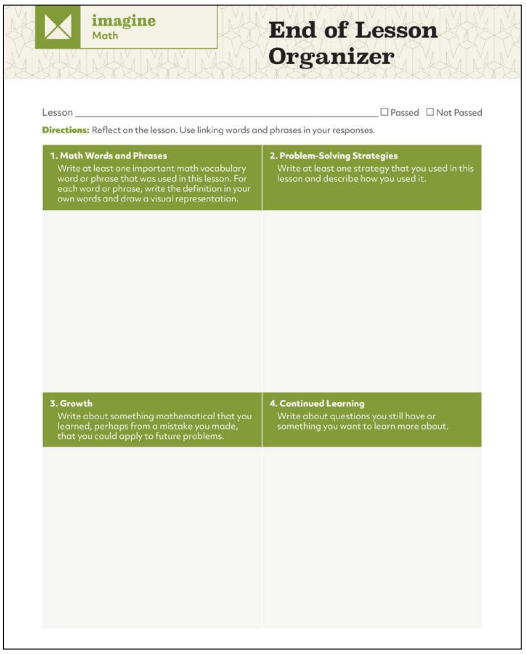

Figure 4. *Journaling pages*

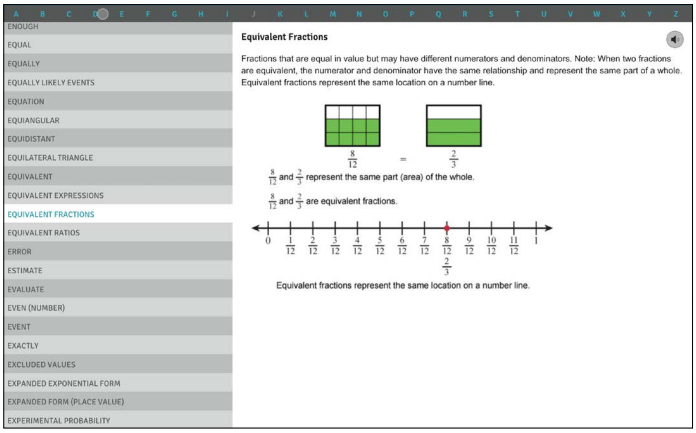

Figure 5. *Interactive glossary*

Conclusion

Imagine Math 3+ promotes mathematical discourse by prompting students to explain and justify their ideas, connect prior knowledge to new concepts, clarify understanding, and communicate their thinking effectively. Throughout the program, students are not only exposed to the language of mathematics, but have opportunity to engage in rich discourse with peers and live tutors that supports them in solidifying conceptual understanding.

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