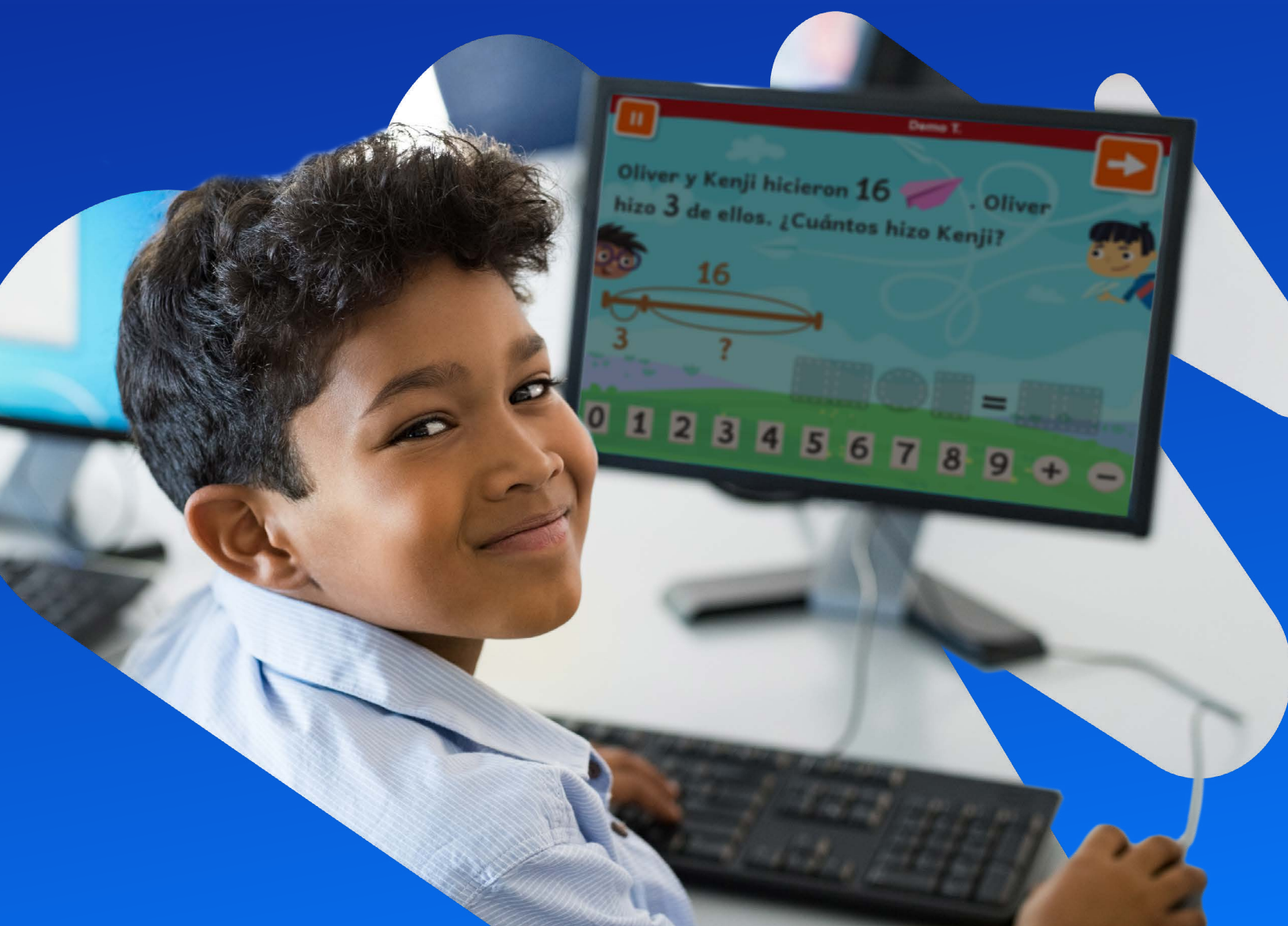


# The Complexities of Math Learning for All Language Learners

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**“...vocabulary in isolation without meaningful context can make it difficult for students to deeply understand and/or apply mathematical words and phrases regularly.”**

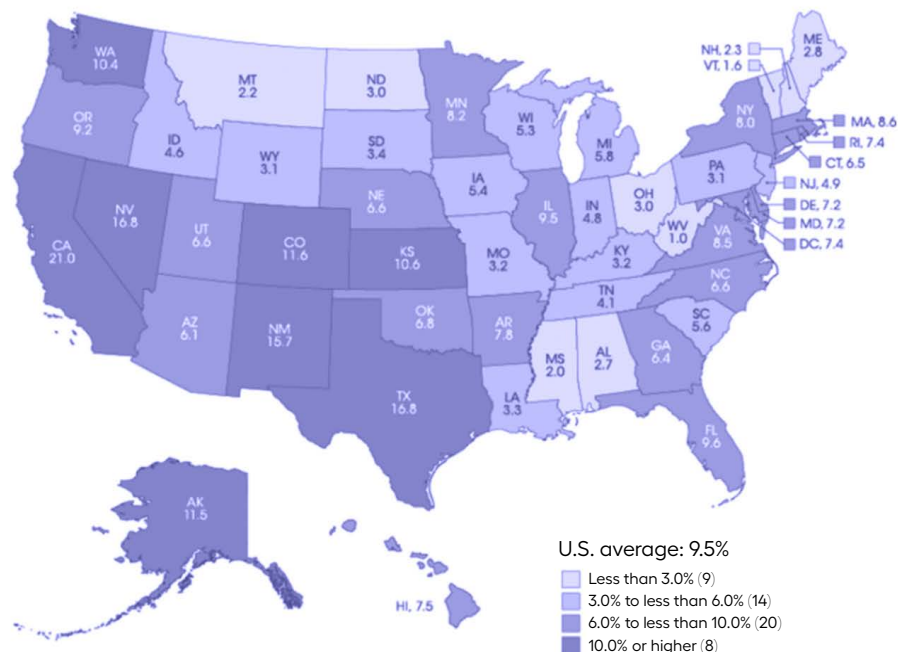
...  
(Hiebert & Grouws, 2007)

## Introduction

There were nearly five million English learners (EL) in US public schools in fall 2015, according to the most recent available data from the National Center for Education Statistics. This represented 9.5% of U.S. public school enrollees, an increase from 8.1% in 2000. Today, educators across the country are seeing an increase in the number of students that are English learners in their classrooms, with almost three out of four American classrooms including at least one English learner (Sparks, 2016).

Many of these schools and districts have put more emphasis on teaching language and literacy skills to ELs rather than focusing on math, as mathematics is thought to be a “universal language” that English learners should do well in. However, educators are finding this is not always the case and mathematics teachers are now charged with teaching math alongside language to help English learner students excel.

**Figure 1:** Percentage of public school students who were English learners, by state as of Fall 2015



Mathematics is often considered to be a universal science of numbers, capable of connecting thinkers across the boundaries of language. Under this premise, EL students would be free from the demands and challenges of language in the mathematics classroom. However, language development and conceptual development are interdependent in mathematics with language playing an integral role in math learning. Mathematics goes beyond numbers, symbols, and operations and students need to use English to access new math concepts, academic language, problem solving, communication and collaboration with peers, which can present challenges for English learners.

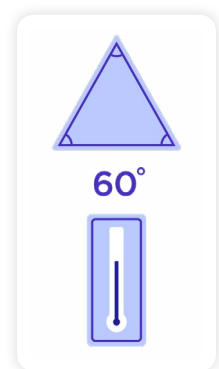
The purpose of this paper is to serve as a practical guide for individuals and/or educators involved in mathematics education of English learners from kindergarten through eighth grade. The resources and evocations provided in this document focus on those educators who directly engage with ELs. Additionally, it is intended to give information to administrators and enable them to better support their educators (teachers) with curriculum and professional development. These strategies can create a supportive and equitable environment that gives English learners the means to achieve math success. The information below is a brief compendium through the complexities of mathematics learning for all language learners noting that when students are learning math, they are also learning English.

## The Role of Vocabulary in Context

Math has a unique language demand for English learners. As a discipline driven by numbers, it is reasonable that many mistake mathematics as a content area with limited language demands that lends itself with relative ease to ELs. Like other subjects, mathematics is linguistically complex and requires students to engage in several language modalities that include reading, writing, listening, speaking, and representing (Celedón-Pattichis & Ramirez, 2012). Students must use all these modalities to understand and apply mathematical ideas.

The mastery of specialized math vocabulary, while not the only language element, plays an essential role for student success in mathematics. Often, vocabulary in isolation without meaningful context can make it difficult for students to deeply understand and/or apply mathematical words and phrases regularly (Hiebert & Grouws, 2007).

The English language itself can pose inimitable challenges along with the hurdles presented by technical mathematical vocabulary. Polysemous terms, or words with more than one meaning, are common in English and can often be encountered in math instruction. For example, “degree” may refer to a unit of measure for an angle or of a temperature. “Degree” may also refer to a type of burn in medicine or severity of a crime in law. For English learners this is one example of a word that may be a stumbling block when unpacking a mathematical task (Molina, 2012).



Homonyms and homophones also create English-specific challenges. For example, a teacher might ask a student to find the “sum” of an extensive list of addends and advise them to add “some” of the numbers in smaller groups as a helpful hint. Words like “base,” “even,” “mean,” and “rational” are further examples that can cause confusion and limit full understanding. This, in turn, can result in the student’s inability to fully grasp a mathematical lesson or concept.

For all students, including English learners, there are instances where common-use English words and phrases may be mathematically imprecise or unclear. For instance, students learning the subtraction algorithm are told to “borrow” which may lead the student to wonder if they must give back what they borrowed somewhere later. Also, when introducing fractions, students may be asked to “reduce” a fraction as opposed to simplifying it, and students might believe that  $\frac{4}{6}$  is a quantity greater than its equivalent  $\frac{2}{3}$  based on the student’s current knowledge or understanding of that word. Thus, students usually rely on what they already know when learning in the math classroom.

English learners, like most students, bring a wide scope of prior knowledge and skillsets to the classroom. With English as the primary language for most instruction, many students may be unaware that they are already familiar with the academic vocabulary within mathematics. The student’s home language coupled with their experiences can be viewed as relevant assets that teachers can use as connections to encourage students to explore new concepts. Research suggests that students show substantial gains when enabled to utilize their native language across content areas to make sense of new concepts (Burchinal, Field, Lopez, Howes & Pianta, 2012; Rodriguez, Duran, Diaz, & Espinosa, 1995; Thomas & Collier, 2003). Leading with a student’s native language can be highly valuable and can benefit teaching English learners new vocabulary across almost any content area (August, Carlo, Dressler, and Snow, 2005, p. 54).

## **“The U.S. educational system has long viewed mathematics and English as separate: Math class is for math; English class is for English”**

For students whose home language is a Romance language (Spanish, Italian, French, Portuguese, and Romanian), cognates (similar origin, i.e. Latin) may be an especially powerful resource educators can influence in mathematics. Spanish, the home language spoken by nearly 80% of English learners (U.S. Department of Education, 2018) shares an estimated 10,000 to 15,000 pairs of cognates with the English language. Cognates comprise over one-third of the words appearing in academic texts. While several of these words are content specific in English, their cognate counterpart in Romance languages are often frequently used and those cognates can provide an effective tool for Spanish speakers when learning in the content area (Dressler, Carlo, Snow, August, & White, 2010).

Through encouragement and support, teachers can help their students develop the habit of looking for cognates in math vocabulary and academic language. Educators can indirectly empower their students to look for and identify the mathematical terms they are familiar with in their native language, enabling them make connections on their own within their learning.

With the recognition of the nuances of the English language and its effects on math learning specific to ELs, educators can lead their students on a path of greater conceptual understanding, given the right resources and support.

Vocabulary Word	Cognate
common base	base común
acute triangle	el triángulo agudo
linear equation	ecuación lineal
geometric figure	figura geométrica
equivalent decimals	decimales equivalentes
concave curve	curva cóncava
vertical distance	distancia vertical
zero divisor	divisor cero

## Developing Oral Language through Mathematical Discourse

Developing oral language via classroom discussion is a resourceful tool that teachers may use to assist with English learners achieving mathematical literacy. If students are going to deeply engage with the language of mathematics, they must have ample opportunities to practice presenting conjectures, evaluating the reasoning of others, and collaborating in the problem-solving process. Therefore, students need opportunities to communicate their ideas that will reinforce and advance EL's overall development of English. Engaging in conversation around mathematics and the academic language that surrounds it will expose students' level of understanding about a mathematical topic as well as the ability to develop arguments.

Focusing on facilitation rather than direct instruction and putting best practices into place, math educators will see classroom discourse flourish. Education researchers Smith and Stein (2018) offer five practices for teachers to engage in to orchestrate productive mathematics discussions:

1. **Anticipate** student responses to challenging mathematics tasks, preparing to address common misconceptions
2. **Monitor** students' responses as they are working on their mathematical problems.
3. **Select** certain students who you would like to present their mathematical work during a whole-class discussion.
4. **Sequence** students' responses to best orchestrate a whole-class discussion so that student responses are purposefully shared.
5. **Connect** different student responses to highlight key mathematical ideas throughout your discussion.



**“The students are the ones who need to learn mathematics, not the teacher; therefore, the students need to be talking, conjecturing, experimenting, thinking, generating, and participating in mathematical discourse”**

Mathematical discourse involves all aspects of mathematical conversation; not just language, but also how students communicate in mathematics (Moschkovich, 2015). Teachers that can model proper conversations with precise mathematical conjectures whenever they pose a new question or idea to their class will see improved sharing of mathematical ideas amongst their students. For example, avoiding imprecise language such as “it,” “that,” and “what” in favor of precise terminology is an important part of modeling precision.

To shape lifelong mathematical practices, students must come to expect that they will always have to work beyond algorithms—that the ability to explain and justify their approach is of equal, if not greater, importance in mathematics. In the classroom, teachers can foster this mindset by using open-ended questioning, exploring mathematical misunderstandings in a positive frame, and facilitating student discussions where students are able to critique, defend, and conclude reasoning in a safe environment.

Incorporating talk moves with English learners is another instrument that teachers can source to make mathematical discourse productive. Educational researchers Chapin, O’Connor and Anderson (2009) created a framework for five talk moves to use with mathematics students detailed below:

1. **Revoicing:** teacher restates a statement as a question to clarify, add more precise vocabulary, and/or involve more students in the conversation. This also can increase comprehension for ELs.
2. **Rephrasing:** teacher asks a student to restate another student’s idea in their own words to give students another way to think about the same concept while involving more students in the conversation.
3. **Reasoning:** teacher asks a student what they think of an idea proposed by another student, such as whether they agree or disagree with another student’s mathematical response.
4. **Elaborating:** teacher asks students to add on, elaborate, or offer an example. This talk move increases classroom involvement in the conversation and can offer extensions.
5. **Waiting:** teacher enables quiet time for students to gather their thoughts before providing an answer.

Students should be reminded that communication is helpful for understanding mathematical concepts. They need reassurance that although it can be unnerving to take part in discussions about mathematics, it is beneficial to their learning.

## Leveraging Multiple Mathematical Models, Manipulatives, and Digital Tools

Mathematics instruction rich with graphics, models, manipulatives, and digital tools can greatly assist all learners with making sense and linking together mathematical concepts. Here, we discuss how to leverage these practices within the classroom to better serve the needs of students.

Graphic representations are an excellent tool for aiding students as they learn vocabulary in mathematics. These pictures can function as a mnemonic device, helping students connect to and easily recall the concept at hand (Celedón-Pattichis & Ramirez, 2012, p. 67). Teachers may strongly encourage the use of math journals, illustrated glossaries, or graphic organizers to help students make these connections. Students can write notes in their native language or, for an added benefit, record these thoughts and ideas in their native language to listen to later. Doing this can enable greater understanding of the concept at hand while increasing students' mathematical literacy.

The use of static visual models and digital mathematical tools can help all students and bring opportunities for authentic and meaningful scaffolds for English learners. The utilization of these tools makes abstract concepts simultaneously concrete and flexible for students. Manipulatives turn math into something tangible that students reshape, reform, and create, ultimately providing students with low-risk experiences to explore and apply reasoning.

Teachers can encourage students to explore multiple representations such as words, drawings, symbols, diagrams, tables, graphs, and/or equations so that all students have opportunities to communicate about mathematical concepts (Moschkovich, 2012, p. 304). The added use of counters, base-ten blocks, paper money, tangrams, algebra tiles, or any other digital, commercial, or homemade manipulative are exceptional tools that give students opportunities to make connections between the real and the abstract (Furner, Yahya & Duffy, 2005).

In addition to hands-on models and manipulatives, new virtual manipulatives and tools allow teachers to scale manipulatives with ease. The National Council of Teachers of Mathematics (2000) describes how technology can deliver powerful visual models, promote conjecturing, and encourage ownership of mathematical ideas. Well-designed virtual manipulatives can increase the diversity of problems that students are engaged with and support the development of abstract representations (Goldenberg, 2000; Reiten, 2018; Sarama & Clements, 2000).

There are other advantages of utilizing virtual manipulatives (Moyer, Bolyard, & Spikell, 2002) such as:

- quickly available for and usable for all students;
- engage students in game-like environment;
- older students connect with them;
- potential personalization of the manipulatives (i.e. changing colors, labels, etc.);

- potential for immediate feedback for student learning allowing students to practice on their own; and
- offer opportunities to scaffold instruction for students to offload mathematical calculations enabling students to focus on new concepts more readily (i.e. calculating slope or plotting points automatically).

The use of virtual manipulatives improves understanding of the abstract language of math while connecting the visual and symbolic representations. Additionally, virtual manipulatives are helpful for all students and students with challenges in language, including English learners.

## **Authentic Problem Solving and the Productive Struggle**

Teachers should explicitly teach students how to problem solve in their mathematics classroom and set high expectations for English learners, giving them mathematical problems that engage students in authentic problem solving. Educators may hesitate to set high demands for ELs, however if English learners are not actively engaged with challenging mathematics, they will suffer in the short and long term (Celedón-Pattichis & Ramirez, 2012). Language needs cannot be confused with an inability to perform mathematically.

Mathematics classrooms, like all classrooms, can be spaces of high-level academic opportunities for all students. The mathematics experience should be driven by rich, authentic tasks with high-levels of cognitive demand. It is essential that mathematical tasks deliver opportunities for students to demonstrate understanding while connecting concepts both to each other and to real-life experiences. In this work, the underlying intention should always

be to encourage deep thinking over the process of obtaining the answer. Additionally, being able to provide multiple points of entry into a problem, a best practice for all students, is a vital differentiation strategy for English learners along with offering challenging problems that encourage struggle.

Productive struggle is sometimes misinterpreted as a directive to create difficult tasks for the sake of the difficulty itself. In mathematics, struggle is synonymous with the meaningful effort to make sense of mathematics. Productive struggle does not involve needless frustration or extreme levels of challenge that result from problems that are nonsensical or overtly difficult. Problems that promote productive struggle are within reach and allow students opportunities to grapple with ideas that are comprehensible yet not entirely formed, leading to a deep and flexible conceptual understanding. Most importantly, tasks which require productive struggle lead students to go beyond memorization or the repetition of a process that has been previously modeled (Hiebert & Grouws, 2006). Students thus spend their time focused on higher-order processes such as analysis or synthesis.



As students wrestle with difficult mathematical concepts and engage in productive struggle, teachers can engage in the following practices to help ELs (Dong, 2016):

- Model the problem-solving process with students;
- Learn about the prior learning and cultural knowledge that English learners bring into the mathematics classroom;
- Set word problems in familiar contexts either by having ELs use their own experiences and/or write their own word problems using familiar context;
- Tap into students previously learned skills; and
- Use language supports and various questioning techniques to help student understanding, such as, translating symbolic math into English.

Through powerful productive struggle, educators can increase student creativity and build authentic problemsolving skills, giving all learners a path to deeper learning.

**An effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics”**

...  
National Council of Teachers in Mathematics, 2014, p. 48.

## Conclusion

The language abilities of English learners can vary from one domain to the next and taking into consideration each student’s unique language needs in the math classroom can be overwhelming. In this paper, we provided educators with some research-based strategies to help English learner students in K-12 mathematics classrooms in relation to: mathematical literacy and vocabulary instruction in context; oral language and mathematical discourse; how to leverage various mathematical models, manipulatives, and digital tools; and authentic problem solving. This information, however, can enrich classroom instruction for all students. In today’s schools, all students are language learners and all teachers are language instructors and language is essential to all learning.

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