White Paper

Imagine IM®

Supporting Meaningful Mathematics Learning for Students with Diverse Learning Needs

Introduction

Across the nation, students with Individualized Education Plans (IEPs) continue to face persistent and widening achievement gaps in mathematics. On the 2024 National Assessment of Educational Progress (NAEP), 74% of eighth grade students with IEPs scored below the Basic level, compared to only 25% of their peers without IEPs (NCES, 2024). These disparities extend beyond standardized testing, with students with IEPs completing fewer advanced math courses and earning lower mathematics achievement throughout their academic careers (Institute of Education Sciences [IES], 2022).

Recent trends underscore the urgency of this challenge. Since 2019, math scores among students with IEPs have declined or remained stagnant (NCES, 2023). Despite decades of policy attention and targeted supports under the Individuals with Disabilities Act (IDEA), students with IEPs continue to score substantially lower on state and national assessments (U.S. Department of Education, 2022).

Importantly, these gaps are not the result of ability, but of opportunity—uneven access to high-quality, evidence-based instruction that meets diverse learning needs to support all learners (Ballin et al., 2022; Connor & Cavendish, 2018; Schulte & Stevens, 2015). Research consistently shows that students with special needs benefit from rigorous grade-level instruction when appropriate supports are in place (Ballin et al., 2002; Connor & Cavendish, 2018). Yet too often, students encounter fragmented instruction focused on isolated procedures rather than deep conceptual understanding, reasoning, and problem solving. Consequently, school districts face a pressing need for mathematics curricula that both uphold rigor and provide built-in supports for diverse learners.

District goals can only be achieved when students have access to rigorous, conceptually rich mathematics curricula grounded in the principles of universal design. Research from cognitive science and special education highlights several conditions that support learning for students with diverse needs: coherent progressions that build understanding over time, scaffolded supports that reduce unnecessary cognitive demands, opportunities for discourse and collaboration, and feedback that promotes reflection and growth. These same principles are reflected in High-Leverage Practices (HLPs) and increasingly characterize high-quality instructional materials (Erdem, 2024; Meyer, Rose & Gordon, 2014; McLeskey et al., 2017; Wijnia et al., 2024).

Imagine IM embodies this vision through its intentional design: scaffolded lessons that maintain grade-level rigor, embedded supports that deepen conceptual understanding, and adaptive feedback that meets students where they are to maintain rigor while expanding access. This paper explores how accessibility and ambition can — and should — coexist to drive equitable outcomes for students with IEPs.

Evidence-Based Conditions for Mathematical Learning

Historically, math instruction has often emphasized repetition and memorization, such as "drill and kill" exercises that reward procedural recall over conceptual understanding, with little peer-to-peer collaboration or interaction (Boaler, 2022; Hierbert & Grouws, 2007). While procedural knowledge is an important component of mathematical proficiency, instruction that focuses primarily on following steps can limit opportunities for students to develop conceptual understanding, explain their reasoning, and apply mathematics flexibly in new situations. In other words, students may learn how to perform operations without understanding why they work.

Research increasingly demonstrates that students develop mathematical understanding when they actively make sense of ideas, connect concepts over time, discuss and justify their reasoning, receive feedback, and apply learning across contexts (Hierbert et al., 1996; NCTM, 2014). Students learn mathematics best when they actively construct understanding, not just memorize procedures (Hierbert et al., 1996; Hierbert & Grouws, 2007; NCTM, 2014). Collectively, this body of work points to four key conditions that support math learning:

- Coherence and connections
- Multiple representations
- Mathematical discourse and collaboration
- Scaffolded support, feedback, and reflection

These conditions are particularly important for students with IEPs, who often benefit from explicit connections between concepts, scaffolded supports, multiple representations, and structured opportunities to generalize learning across contexts. These principles closely align with the High-Leverage Practices (HLPs) identified by the Council for Exceptional Children as essential for supporting students with special needs (McLeskey et al., 2017). Together, mathematics education and special education research converge to a common conclusion: students learn best when instruction is coherent, collaborative, accessible, and intellectually rigorous.

COHERENCE AND CONNECTIONS

Intentional connections between mathematical ideas, prior knowledge, and real-world experiences are foundational to deep mathematical learning. New learning is strengthened when concepts are introduced through coherent progressions that build on what students already know rather than as isolated skills or procedures (Hiebert et al., 1996; NCTM, 2014). As students make connections across concepts and experiences, they develop a relational understanding of mathematics, understanding not only how mathematical procedures work, but why they work (Hierbert & Carpenter, 1992; Skemp, 1976).

Rather than learning a procedure for a single type of problem, students learn underlying concepts and relationships that can be applied across situations. Because concepts build logically on one another, students can draw on prior knowledge when encountering new problems rather than treating each problem type as entirely new. Learning progressions strengthen this process by revisiting and extending ideas over time, allowing students to apply earlier understandings in increasingly sophisticated ways (Confrey et al., 2014; Daro et al., 2011). As a result, rather than obtaining a collection of disconnected rules, students develop an interconnected system of mathematical ideas and knowledge that can be flexibly applied across problems and contexts. This connected understanding enables students to select strategies, make connections among concepts, and adapt their thinking to new contexts when solving unfamiliar problems (Bransford et al., 2000; Kilpatrick et al., 2001).

When mathematics is presented as a series of disconnected topics, each new unit can feel like starting over, creating uncertainty and undermining confidence. In a coherent curriculum, students can rely on existing mental models to understand new concepts, reducing frustration and anxiety that often accompany learning isolated new procedures. This predictability allows students to focus on reasoning and sense-making rather than focusing on disconnected rules (Paas et al., 2003). By supporting deeper understanding and transfer, coherent instruction can strengthen students' mathematical confidence and willingness to engage with challenging problems (Boaler, 2016; Hiebert et al., 1997).

For students with IEPs, coherent progressions are especially important because they support the maintenance and generalization of learning, both of which are emphasized in High-Leverage Practices (McLeskey et al., 2017). Many students with learning disabilities, executive functioning challenges, processing difficulties, or working memory limitations benefit from consistent structures, explicit connections, and opportunities to revisit important concepts over time (Gersten et al., 2009; Swanson & Beebe-Frankenberger, 2004). Moreover, this structure makes it easier to provide targeted supports, identify misconceptions, and connect intervention efforts to core instruction (Powell et al., 2020; Riccomini et al., 2015). Through coherent learning progressions, students are better able to retain, transfer, and apply mathematical understanding in increasingly complex situations (Fuchs et al., 2008; Powell & Fuchs, 2015).

MULTIPLE REPRESENTATIONS

Many mathematical ideas are inherently abstract. Multiple representations help make these ideas visible by expressing the same concept through concrete, visual, verbal, symbolic, and numerical forms (Fyfe et al., 2014; NCTM, 2014). Because each representation highlights different features of a concept, students gain access to mathematical relationships that may be difficult to recognize through symbolic notation alone. Students can use representations to test ideas, identify patterns, make predictions, and justify conclusions.

Representations are particularly powerful because they reveal mathematical structure. For example, a ratio may be represented through a table, double number line, graph, equation, or verbal description. While each representation communicates the same relationship, each draws attention to different mathematical features and patterns. In this way, representations do more than communicate information, they provide tools for reasoning. Deep understanding emerges when students begin to move between representations and recognize how each illustrates the same underlying concept (Fyfe et al., 2014; National Research Council, 2001).

For students with IEPs, multiple representations provide multiple entry points into learning, serving as powerful tools for both learning and communication. Visual models, manipulatives, diagrams, and graphic representations make mathematical thinking visible, allowing students to reason about relationships that may be difficult to express through language or symbols alone (Fyfe et al., 2014; King et al., 2002; Swanson & Beebe-Frankenberger, 2004). Because representations make student thinking visible, they also provide teachers with insight into students' reasoning, misconceptions, and developing understanding (Gersten et al., 2009; King et al., 2002). Consistent with High-Leverage Practices that emphasize scaffolded supports, multiple representations increase access to rigorous mathematical ideas without reducing expectations for learning.

MATHEMATICAL DISCOURSE AND COLLABORATION

Mathematics is not only a discipline of numbers and symbols; it is also a discipline of communication. Mathematical discourse is more than simply talking about math; it is a set of tools and practices that make learning visible. Engaging in mathematical discourse helps students clarify understanding, identify misconceptions, and refine their ideas (Mercer & Howe, 2012; NCTM, 2014; Vygotsky, 1978; Xin et al., 2019).

Research grounded in sociocultural theories of learning suggests that knowledge is constructed through interaction with others (Vygotsky, 1978). As students collaborate and engage in discussion, they develop both conceptual understanding and mathematical language (Aguirre & Bunch, 2012; Moschkovich, 2015). The teacher's role shifts from delivering information to facilitating inquiry, guiding discussion, and helping students make connections among ideas. In this way, discourse transforms the classroom into a community of learners where students actively contribute to the construction of knowledge.

Mathematical discourse is particularly important for students with IEPs because it provides opportunities to rehearse language, process ideas aloud, and receive immediate feedback from teachers and peers. Structured discourse routines help students refine understanding while building confidence as mathematical thinkers (Xin et al., 2019). These collaborative experiences align closely with High-Leverage Practices that emphasize student engagement, flexible grouping, and meaningful participation, positioning all learners as valued contributors whose ideas and experiences enrich the mathematical community (Council for Exceptional Children & CEEDAR Center, 2024; McLeskey et al., 2017).

Importantly, when mathematical discourse is intentionally structured and scaffolded, it can be accessible even for students who experience language, communication, or processing challenges. Research suggests that language development and mathematical understanding are deeply interconnected (Aguirre & Bunch, 2012; Moschkovich, 2015). Supports such as sentence frames, partner discussions, visual representations, think-pair-share routines, and guided questioning allow all students to participate meaningfully in mathematical conversations while reducing linguistic demands. Rather than serving as a barrier, well-designed discourse becomes a powerful support for learning by making thinking visible and providing multiple ways for students to communicate mathematical ideas.

SCAFFOLDED SUPPORT AND FEEDBACK

Learning improves when teachers provide supports that help students engage in challenging work while gradually increasing independence (CAST, 2018; Hiebert & Grouws, 2007; King et al., 2022). Through scaffolding, complex concepts and tasks are made more accessible without reducing the rigor of the instruction. Effective scaffolds may include visual models, guiding questions, sentence frames, worked examples, manipulatives, or structured routines that help students focus on the key mathematical ideas.

Scaffolding also helps students manage cognitive demands. When students are asked to manage too many steps or ideas at once, working memory can become overwhelmed, making it harder to focus on the underlying mathematics (Paas & Sweller, 2012). Strategic supports break learning into manageable parts, make relationships more visible, and help students build understanding step by step. As students gain confidence and competence, these supports can be gradually faded, promoting increasing independence (CAST, 2018; King et al., 2022).

Feedback and reflection deepen this learning process. Constructive feedback helps students identify misconceptions, refine strategies, and understand how to improve their reasoning. Reflection gives students opportunities to monitor their understanding, evaluate their strategies, and make sense of what they learned (Mercer & Howe, 2012; NCTM, 2014). Together, feedback and reflection support metacognition, helping students think not only about the answer, but about the reasoning and decision-making processes used to arrive at it (McLeskey et al., 2017; Wijnia et al., 2024).

For students with IEPs, scaffolded support, feedback, and reflection are especially important because they provide access to rigorous grade-level work without lowering expectations. When implemented intentionally, scaffolds do not simplify mathematics. Instead, they create the conditions students need to engage with complex ideas and develop independence as learners.

From Research to Practice: How Problem-Based Instruction Creates the Conditions for Successful Mathematical Learning

Increasingly, high-quality instructional materials (HQIMs) in mathematics are characterized by their emphasis on deep conceptual understanding, mathematical reasoning, and student sense-making rather than procedural recall alone (EdReports, 2025; NCTM, 2014). This shift reflects a growing consensus that students develop durable mathematical understanding when they actively construct knowledge, engage in productive struggle, and make connections among mathematical ideas (Erdem, 2024; Kilpatrick et al., 2001). These principles align with several High-Leverage Practices identified by the Council for Exceptional Children, including maintaining and generalizing learning, providing scaffolded supports, teaching cognitive and metacognitive strategies, using flexible grouping, and delivering constructive feedback (McLeskey et al., 2017).

As a result, many of the most highly regarded mathematics curricula organize instruction around problem-based learning experiences that require students to grapple with meaningful mathematical tasks, discuss strategies, justify reasoning, and make connections among ideas. In this way, Problem-Based Instruction (PBI) is not separate from high-quality mathematics instruction. Rather, PBI has become a defining feature of many contemporary mathematics programs designed to support conceptual understanding and long-term mathematical proficiency (Erdem, 2024; Wijnia et al., 2024).

PBI brings together coherence, multiple representations, discourse, collaboration, scaffolding, feedback, and reflection by organizing learning around meaningful mathematical tasks. Rather than memorizing isolated procedures, students learn mathematics by doing mathematics: reasoning through problems, discussing strategies, making connections, and constructing understanding with others (Hmelo-Silver, 2004; Hung et al., 2008).

For students with IEPs, this approach can be a powerful equalizer. PBI provides multiple entry points into rigorous content, supports the maintenance and generalization of learning, and creates structured opportunities for collaboration, feedback, and reflection. By embedding supports within rich mathematical experiences rather than separating students from them, PBI helps ensure that all learners can participate meaningfully in grade-level mathematics while developing confidence, competence, and independence.

In a typical PBI lesson, teachers introduce a meaningful problem and establish a shared understanding of the task. Students then work individually or in small groups to explore solutions while teachers provide strategic guidance, pose questions, and highlight important connections. Class discussions bring student strategies to the surface, allowing learners to compare approaches, refine reasoning, and build shared understanding. This structure positions every student as a capable problem-solver and values diverse approaches to thinking.

For students with IEPs, multiple access points are embedded throughout the lesson. Students may engage with a problem through visual representations, manipulatives, diagrams, numerical reasoning, discussion with peers, or teacher-guided questioning. Because problems can be approached using different strategies, students are able to enter the mathematics from their own strengths while engaging with the same rigorous content. These opportunities for participation align closely with Universal Design for Learning principles and High-Leverage Practices that emphasize access, engagement, and meaningful participation (CAST, 2018; McLeskey et al., 2017).

PBI also emphasizes coherence over coverage. Rather than moving quickly through disconnected procedures, lessons focus on helping students understand the relationships among concepts, strategies, and representations. Teachers intentionally surface connections and revisit important ideas over time, allowing students to build deeper understanding and generalize learning across contexts. For students with IEPs, who often experience fragmented instruction or reduced expectations, this coherent approach can be a powerful equalizer in supporting retention, transfer, and application of learning (Gersen et al., 2009; Powell & Fuchs, 2015).

PBI also supports executive functioning and reduces unnecessary cognitive load. Predictable instructional routines, collaborative problem solving, visual representations, and structured discussion protocols help students manage complex tasks without relying solely on working memory (Cragg & Gilmore, 2014; Swanson & Beebe-Frankenberger, 2004). By organizing learning around mathematical structures and relationships, PBI helps students build schemas that can be applied across contexts. Techniques such as concrete-representational-abstract (CRA) sequences, worked examples, and explicit strategy cues support understanding while offloading cognitive demand (Fyfe et al., 2014). Guided PBI, when structured intentionally, keeps learning within students' cognitive reach while fostering independence, collaboration, and problem-solving skills and valuing the perspectives of all students.

In short, PBI levels the playing field for students with IEPs by providing authentic entry points into rigorous mathematics, supporting executive functioning through structured discourse and visual reasoning, and creating opportunities for meaningful participation in mathematical communities. When implemented through high-quality, scaffolded materials such as Imagine IM, PBI becomes a tool not only for engagement but for genuine access to mathematical understanding.

What is Imagine IM?

Imagine IM is a certified Illustrative Mathematics program from Imagine Learning, offering the most up-to-date IM-certified curriculum and content, optimized for engagement, accessibility, and usability. As a premium partner, Imagine Learning embodies the rigor and integrity of the IM curriculum with a keen focus on engagement, usability, and accessibility. The comprehensive print and digital K–12 math program is classroom ready and enhanced with exclusive features to make instruction come alive. It transforms mathematics classrooms into inclusive learning communities full of curiosity, collaboration, and enjoyment.

Imagine IM is grounded in the belief that all students are capable of learning mathematics. Each unit and lesson is intentionally designed to support meaningful access to grade-level content through equitable classroom structures and research-based teaching practices. Lessons begin with a clear learning goal and warm-up, followed by problem-based activities that invite students to reason, explore, and make sense of ideas in collaboration with their peers. Each lesson concludes with synthesis and reflection, allowing students to consolidate their understanding while teachers gather insight into student thinking. Units begin with an Invitation to Mathematics, offering accessible entry points that help teachers observe students' prior understandings — an important equalizer for students with IEPs.

Inclusive Design and Supports for Students with IEPs

Imagine IM's design is rooted in several guiding principles that align with the National Council of Teachers of Mathematics (NCTM) and Council for Exceptional Children (CEC) position statements on inclusive mathematics instruction (NCTM, 2014):

- All students can learn meaningful mathematics when given access and support.
- Mathematics classrooms must presume competence and position every learner as a valued, capable participant.
- Learning happens by doing. Students develop understanding through active problem solving, not passive repetition.

Collectively, these principles operationalize key SPED priorities by supporting instruction in the least restrictive environment (LRE) and ensuring that students with IEPs have meaningful access to grade-level mathematics. By embedding supports directly within core instruction, Imagine IM strengthens Tier I instruction within a Multi-Tiered System of Supports (MTSS), helping address learner variability proactively rather than through remediation.

LEARNING BY DOING

Students learn mathematics by doing mathematics. Rather than memorizing procedures, they engage in mathematical practices: making sense of problems, reasoning abstractly, constructing arguments, and modeling with mathematics (Hiebert et al., 1996). These experiences allow students to see themselves as mathematical thinkers with ideas worth sharing. For students with IEPs, this "learn-by-doing" structure provides multiple access points and authentic opportunities to engage with mathematical reasoning, reducing dependence on rote recall and supporting deeper understanding.

BUILDING A MATHEMATICAL COMMUNITY

Community is central to learning and identity development (Hammond, 2015; Vygotsky, 1978). Imagine IM lessons foster rich mathematical discourse, helping students articulate and refine their thinking in collaboration with others. Teachers are supported with discourse routines and participation structures that increase reasoning opportunities for students with learning disabilities. The curriculum's early units explicitly establish community norms, build shared trust, and create an environment where every student's contribution is valued. In this way, Imagine IM positions inclusion not as presence, but as participation.

MODELING AND REPRESENTATION

In the elementary grades, Imagine IM emphasizes modeling with mathematics, encouraging students to use drawings, manipulatives, tables, and equations to represent ideas. These multiple representations align with the Concrete–Representational–Abstract (CRA) sequence shown to support students with learning difficulties. Modeling naturally externalizes thinking, offloading cognitive demand and allowing students with working-memory challenges to focus on reasoning rather than recall.

Universal Design for Learning (UDL) in Practice

Imagine IM embeds inclusive design at every level of instruction. Each lesson includes Access for Students with Diverse Abilities, a dedicated section providing flexible supports to increase access, reduce barriers, and maximize learning. These supports are grounded in the Universal Design for Learning (UDL) framework (CAST, 2018), offering multiple means of engagement, representation, and action/expression. Teachers can adjust supports to address specific cognitive functions or learning needs—without lowering the mathematical rigor of the task — and fade them out as students gain independence.

The curriculum also integrates language development as a central component of mathematical understanding. Mathematics is inherently language-rich: students must read, write, listen, and speak to make sense of problems and communicate reasoning (Aguirre & Bunch, 2012). Imagine IM advances both math and language learning through four design principles:

1. Support sense-making – Scaffold tasks and amplify (not simplify) language so all students can make meaning.
2. Optimize output – Create frequent, supported opportunities for students to articulate mathematical ideas orally, visually, and in writing.
3. Cultivate conversation – Foster constructive mathematical discourse that builds shared understanding (Mercer & Howe, 2012; Zwiers, 2011).
4. Maximize meta-awareness – Encourage students to reflect on their reasoning and communication, developing metacognitive and metalinguistic skills.

By integrating these supports within core instruction, Imagine IM ensures that students with IEPs and other learning differences can engage meaningfully with grade-level mathematics — not through simplification, but through accessibility. The curriculum's combination of problem-based inquiry, visual modeling, discourse, and language-rich routines empowers all students to see themselves as capable mathematicians.

Research Evidence: Closing the gap

Across diverse contexts, recent evaluations have shown that Imagine IM is helping schools make measurable progress in narrowing long-standing performance gaps between students with and without IEPs.

In a 2025 quasi-experimental study conducted by the Johns Hopkins University Center for Research and Reform in Education (JHU CRRE), K–1 students with IEPs using Imagine IM scored an average of 8.5 points higher on FastBridge earlyMath — a nationally normed measure of early numeracy — than their peers in matched comparison classrooms (effect size = 0.40 SD). In Grade 3, students with IEPs achieved 1.4 points higher on FastBridge aMath, a computer-adaptive assessment of mathematics achievement (effect size = .12 SD), reflecting statistically and practically meaningful gains.

Complementary findings emerged from an evaluation of California 2025 statewide CAASPP math performance. This study examined outcomes across 38 districts in California that implemented Imagine IM in Grades K–12. After controlling for prior performance and demographics, students with IEPs in Imagine IM schools scored, on average, 4.5 points higher on the CAASPP Math assessment than peers in non-IM schools, with more students with IEPs meeting or exceeding standards (+1.2 percentage points). Importantly, students with IEPs showed a larger gain from Imagine IM than their non-IEP peers, reducing the achievement gap by roughly three scale-score points.

Importantly, both the Johns Hopkins University and California statewide analyses meet ESSA Tier III ("Promising") evidence standards, meaning they employed rigorous research designs and the observed results demonstrate statistically meaningful improvements for a defined subgroup — in this case, students with IEPs. This designation affirms Imagine IM as an evidence-based intervention under federal accountability frameworks, including ESSA and IDEA, and highlights its effectiveness in supporting diverse learners within core instruction. For districts, these findings translate into clear alignment with policy priorities: Imagine IM is a research-supported core curriculum that improves outcomes for students with IEPs while maintaining access to grade-level mathematics.

Together, these studies provide converging evidence that Imagine IM is an effective, equity-centered mathematics program that advances both access and outcomes. By combining problem-based learning with built-in supports grounded in Universal Design for Learning (UDL) and Multi-Tiered Systems of Support (MTSS), Imagine IM enables all students — especially those with IEPs — to participate fully, think deeply, and achieve at higher levels.

Conclusion

The evidence is clear: closing opportunity gaps in mathematics requires more than additional time or remediation; it requires access to meaningful, conceptually rich learning experiences that honor every student's potential. Imagine IM exemplifies this approach. By combining rigorous, standards-aligned content with intentional design for inclusion, the program enables students with IEPs and their peers to engage in authentic mathematical reasoning, discourse, and problem solving.

Across independent evaluations, Imagine IM has demonstrated measurable progress in raising achievement and narrowing gaps for students with IEPs. These gains reflect more than test score improvements; they signal shifts in access, expectations, and belonging. When students with IEPs participate fully in mathematical inquiry, share their reasoning, and see their ideas valued, achievement follows naturally.

For districts, these findings offer a path forward: invest in curriculum models that blend rigor with accessibility and professional learning that empowers teachers to implement inclusive practices with fidelity. As schools continue to recover from pandemic-era learning disruptions and rebuild toward equity, Imagine IM provides a model of what's possible: mathematics classrooms where all students are seen, supported, and successful.

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